UNCLASSIFIED

AD NUMBER
AD018509
CLASSIFICATION CHANGES
TO: **unclassified**
FROM: **confidential**
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; AUG 1953. Other requests shall be referred to Office of Naval Research, Arlington, VA 22203.**
AUTHORITY
ONR ltr, 10 Sep 1954; ONR ltr, 9 Nov 1977

THIS PAGE IS UNCLASSIFIED

UNCLASSIFIED

AD NUMBER
AD018509
CLASSIFICATION CHANGES
TO: unclassified
FROM: confidential
AUTHORITY
ONR ltr, 10 Sep 1954

THIS PAGE IS UNCLASSIFIED

CONFIDENTIAL
Security Information

THE TWO-DIMENSIONAL WAKE AND DOWNWASH OF A HYDROFOIL IN WAVES

PREPARED UNDER
U.S. NAVY
OFFICE OF NAVAL RESEARCH
CONTRACT Nonr 263-01
(E.T.T. PROJECT NO. DZ1414)

REPORT NO. 497

August 1953

Prepared by: Paul Kaplan
Paul Kaplan

Approved by: WC Hugli Jr
W.C. Hugli, Jr.

EXPERIMENTAL TOWING TANK
STEVENS INSTITUTE OF TECHNOLOGY
HOBOKEN, NEW JERSEY

Security Information
CONFIDENTIAL

TABLE OF CONTENTS

SUMMARY

The downwash angle and its rate of change with foil angle are derived for a hydrofoil moving under a free surface disturbed by sea waves. By means of a linearized analysis, the downwash and its rate of change are found to be the sum of the values determined for motion under an undisturbed free surface and a sinusoidal time-dependent component due to the sea waves. The phase angle and frequency relationships for the time-dependent components are found for both following-sea and head-sea conditions.

INTRODUCTION

In order to study the stability of a tandem hydrofoil system in waves, it is necessary to determine the forces acting at the rear foil and their derivatives. The rear foil will be in a flow field composed of the wave system through which the configuration moves and of the wake of the forward foil. Since the force derivatives for the rear foil are affected by the downwash through inclusion of terms involving $d\epsilon/d\alpha$ (rate of change of downwash angle with respect to angle of attack of the forward foil) in the equations of motion (Reference 1), the necessity of determining the downwash pattern is evident.

In the present report, an analysis is made of the effect of ocean waves on the lift force of a single hydrofoil restrained in heave and pitch, based on the classical wave theory which assumes a sinusoidal wave form. The resultant wake disturbance generated by the hydrofoil (time-dependent at any point located at a constant distance aft of the foil), the downwash angle ϵ, and $d\epsilon/d\alpha$ are then determined. The resultant water surface shape at any distance aft of the foil is also found in this analysis, giving a representation of the wave pattern in which the second foil of a tandem system will be operating.

The study was carried out at the Experimental Towing Tank, Stevens Institute of Technology, under Office of Naval Research Contract No. Nonr 263-01.

SYMBOLS

A = aspect ratio
a = surface wave amplitude
C_{L_S} = quasi-steady lift coefficient
C_{L_T} = total lift coefficient in waves
C_{ℓ_S} = quasi-steady two-dimensional lift coefficient
c = wave velocity
c' = mean hydrofoil chord
g = acceleration due to gravity
H = vertical displacement of hydrofoil from equilibrium, positive downwards
h = submergence of foil below smooth water surface
s = distance aft of 1/4-chord point of hydrofoil
t = time
u_o = longitudinal component of orbital velocity
V = velocity of foil
w_o = vertical component of orbital velocity
$\hat{w}_o$ = w_o/V, change in angle of attack due to orbital velocity
α = hydrofoil angle of attack
γ, γ' = circular frequency of encounter in following sea and head sea, respectively
ϵ = downwash angle
ζ_s = disturbance wave produced in smooth water
ζ_w = disturbance wave produced in waves
η = sea wave surface displacement
η_r = resultant wave surface displacement aft of hydrofoil
λ = wave length of sea waves
$\phi, \psi, \delta, \sigma, \vartheta$ = phase angles

Subscript e denotes equilibrium condition

BASIC ASSUMPTIONS UNDERLYING THE THEORY

This study is limited to the motion of a two-dimensional hydrofoil restrained in heave and pitch and moving at constant speed under a free surface disturbed by sea waves. A linearized treatment is utilized in considering the total lift to be the sum of the quasi-steady lift due to motion at a fixed geometric angle of attack, the lift variation due to the orbital velocities of the waves, and the lift variation due to change in submergence, within the limits of the region of stall. The forces are assumed to respond instantaneously to the wave influence. No consideration is given to the effect on the foil lift of the vortices cast off in the wake due to the nonstationary flow. This assumption is based on the fact that the frequency is low and is not comparable in magnitude to flutter frequencies. Thus, for stability analysis purposes, the unsteady flow effects can be neglected.

PROPERTIES OF THE WAVE MOTION

The wave shape for a simple harmonic progressive wave moving from left to right, and defined with respect to axes fixed in the fluid, is given by

$$\eta = a \sin \frac{2\pi}{\lambda} (X - ct) \quad . \tag{1}$$

The origin of coordinates $X = 0$, $Y = 0$ is taken at the undisturbed water surface, and $t = 0$ is taken at the instant the nodal point passes the origin. The distance X is positive in the right-hand direction and Y is positive upwards.

In a progressive harmonic wave, the fluid particles move in circular orbits and the horizontal and vertical components of orbital velocity and orbital acceleration at a depth h are

$$\left.\begin{aligned} u_o &= a\left(\frac{2\pi c}{\lambda}\right) e^{-2\pi h/\lambda} \sin\frac{2\pi}{\lambda}(X - ct) \quad , \\ w_o &= -a\left(\frac{2\pi c}{\lambda}\right) e^{-2\pi h/\lambda} \cos\frac{2\pi}{\lambda}(X - ct) \quad , \end{aligned}\right\} \tag{2}$$

$$\left.\begin{aligned} \dot{u}_o &= -a\left(\frac{2\pi c}{\lambda}\right)^2 e^{-2\pi h/\lambda} \cos\frac{2\pi}{\lambda}(X - ct) \quad , \\ \dot{w}_o &= -a\left(\frac{2\pi c}{\lambda}\right)^2 e^{-2\pi h/\lambda} \sin\frac{2\pi}{\lambda}(X - ct) \quad . \end{aligned}\right\} \tag{3}$$

The absolute magnitude of the orbital velocity is then

$$|V_o| = a\left(\frac{2\pi c}{\lambda}\right) e^{-2\pi h/\lambda} \quad . \tag{4}$$

For a wave propagating from right to left, the above equations for wave profile, orbital velocities, and orbital accelerations are valid, provided the sign of the wave profile velocity, c, is changed.

TOTAL LIFT VARIATION IN WAVES

The total lift developed by the hydrofoil is equal to the sum of the quasi-steady lift of the foil moving under an undisturbed free surface, the change in lift due to orbital motion, and the change in lift due to the change in submergence of the foil.

The quasi-steady lift coefficient for a two-dimensional hydrofoil is given in Reference 2 as

$$C_{\mathfrak{L}_s} = 2\pi a_g (P_1 - a_g P_2) \quad , \tag{5}$$

where a_g is the geometric setting relative to the smooth water surface (two-dimensional angle of attack), and P_1 and P_2 are dimensionless expressions dependent on depth and Froude number V^2/gc', which express the effect of the proximity of the free surface. For the small angles usually experienced in the motion, the two-dimensional lift coefficient may be approximated by

$$C_{\mathfrak{L}_s} = 2\pi a_g P_1 \quad . \tag{6}$$

For the purpose of applying the results of this investigation to stability studies, a more reliable effect can be expected by utilizing the expressions for lift coefficient and its derivatives that are given by three-dimensional considerations. Thus, the resulting wake disturbance will then be connected with the forces produced by an actual three-dimensional hydrofoil. For an actual three-dimensional foil, it has been determined in Reference 3, by means of the momentum theory for airfoils, that

$$\frac{\partial C_{L_s}}{\partial a} = 2\pi P_1 \frac{\pi A + 8h/c'}{\pi A + (8h/c') + 4\pi P_1} \tag{7}$$

and hence

$$C_{L_s} = 2\pi a P_1 \frac{\pi A + 8h/c'}{\pi A + (8h/c') + 4\pi P_1} \quad , \tag{8}$$

where a in this case is the actual angle of attack of the foil relative to the zero lift angle.

The form of C_{L_s} given by equation (8) may now be used to determine the derivative of lift coefficient with respect to depth, which is found to be

$$\frac{\partial C_{L_s}}{\partial h} = 2\pi a \left\{ \frac{\pi A + 8h/c'}{\pi A + (8h/c') + 4\pi P_1} \frac{\partial P_1}{\partial h} + P_1 \frac{(32\pi P_1/c') - 4\pi \frac{\partial P_1}{\partial h} (\pi A + 8h/c')}{[\pi A + (8h/c') + 4\pi P_1]^2} \right\} . \quad (9)$$

Since, in the linearized analysis, the coefficients are to be evaluated at the equilibrium position and since the definition of H is deviation from equilibrium depth, the relation

$$\left(\frac{\partial C_{L_s}}{\partial h}\right)_e = \left(\frac{\partial C_{L_s}}{\partial H}\right)_{H=0} \quad (10)$$

readily follows. The above derivations may also be found in Reference 1.

The major effect of the orbital motion on the lift of a hydrofoil is to change the angle of attack. The angular change in linearized form is expressed as

$$\hat{w}_o = \frac{w_o}{V} \quad , \quad (11)$$

where V is the forward speed of the hydrofoil. By ignoring the lift variation due to orbital accelerations and the change in longitudinal speed (orbital velocity u_o), the lift variation due to orbital motion may be approximated by

$$\left(\frac{\partial C_{L_s}}{\partial \alpha}\right)\hat{w}_o \quad , \quad (12)$$

as shown in Reference 1.

The change in submergence occurs because of the local elevation or depression of the fluid surface due to the wave motion. The change in lift coefficient due to change in submergence of the foil is then expressed as

$$\left(\frac{\partial C_{L_s}}{\partial h}\right)\eta \quad . \quad (13)$$

The total lift coefficient for the hydrofoil in waves is now given by

$$C_{L_T} = C_{L_s} + \left(\frac{\partial C_{L_s}}{\partial \alpha}\right)\hat{w}_o + \left(\frac{\partial C_{L_s}}{\partial h}\right)\eta \quad . \quad (14)$$

WAKE DISTURBANCE PRODUCED BY THE HYDROFOIL

In studying the flow about a body moving with velocity V with respect to the undisturbed fluid, it is more convenient to refer to axes fixed in the body rather than in the fluid. The required transformation is

$$\left.\begin{aligned} x &= X - Vt \\ y &= Y \\ t &= t \end{aligned}\right\} \qquad (15)$$

where the origin $x = 0$ is taken at the quarter-chord point of the hydrofoil and x is positive to the right (in the direction of motion). The wave profile and orbital vertical velocity are then given as

$$\left.\begin{aligned} \eta &= a \sin \frac{2\pi}{\lambda}[x + (V - c)t] \\ w_o &= -a\left(\frac{2\pi c}{\lambda}\right)e^{-2\pi h/\lambda} \cos \frac{2\pi}{\lambda}[x + (V - c)t] \end{aligned}\right\} \qquad (16)$$

for a following sea and as

$$\left.\begin{aligned} \eta &= a \sin \frac{2\pi}{\lambda}[x + (V + c)t] \\ w_o &= a\left(\frac{2\pi c}{\lambda}\right)e^{-2\pi h/\lambda} \cos \frac{2\pi}{\lambda}[x + (V + c)t] \end{aligned}\right\} \qquad (17)$$

for a head sea. The frequency of encounter for following seas and head seas is given by $(V - c)/\lambda$ and $(V + c)/\lambda$, respectively.

For a hydrofoil moving at speed V and situated at depth h under an undisturbed free surface, the resultant surface wave disturbance (beyond the first 1/4 wave length downstream) is given in Reference 5 as

$$\zeta_s = -C_{L_s} c' e^{-gh/V^2} \sin \frac{gs}{V^2} \qquad , \qquad (18)$$

where s is the distance measured aft of the hydrofoil $(s = -x)$. Since the wave disturbance is proportional to the foil lift coefficient, it has been assumed that the same effect is true for the foil in waves where the lift has a time-dependent sinusoidal variation. Thus, the wake disturbance produced by the hydrofoil operating in waves will also have a time-dependent sinusoidal variation and is given by

$$\zeta_w = -C_{L_T} c' e^{-gh/V^2} \sin \frac{gs}{V^2}$$

$$= -\left[C_{L_s} + \left(\frac{\partial C_{L_s}}{\partial \alpha}\right)\hat{w}_o + \left(\frac{\partial C_{L_s}}{\partial h}\right)\eta\right] c' e^{-gh/V^2} \sin \frac{gs}{V^2} \quad . \tag{19}$$

Substituting the expressions for $\hat{w}_o$ and η from equations (11), (16), and (17) gives the total lift coefficient in a following sea:

$$C_{L_T} = C_{L_s} + a\sqrt{\left(\frac{\partial C_{L_s}}{\partial h}\right)^2 + \left(\frac{2\pi c}{\lambda V}\right)^2 e^{-4\pi h/\lambda}\left(\frac{\partial C_{L_s}}{\partial \alpha}\right)^2} \sin\left[\frac{2\pi}{\lambda}(V - c)t - \phi\right] \quad , \tag{20}$$

where

$$\phi = \tan^{-1} \frac{\left(\frac{2\pi c}{\lambda V}\right) e^{-2\pi h/\lambda}\left(\frac{\partial C_{L_s}}{\partial \alpha}\right)}{\frac{\partial C_{L_s}}{\partial h}} \quad . \tag{21}$$

There is no dependence on the coordinate x since the lift acts at the quarter-chord point of the foil, where $x = 0$. Equation (20) may be simplified by letting

$$\left.\begin{aligned} B &= \sqrt{\left(\frac{\partial C_{L_s}}{\partial h}\right)^2 + \left(\frac{2\pi c}{\lambda V}\right)^2 e^{-4\pi h/\lambda}\left(\frac{\partial C_{L_s}}{\partial \alpha}\right)^2} \\ \gamma &= \frac{2\pi}{\lambda}(V - c) \end{aligned}\right\} \tag{22}$$

where γ is now a circular frequency of encounter, resulting in the form

$$C_{L_T} = C_{L_s} + aB \sin(\gamma t - \phi) \quad . \tag{23}$$

Similarly, for the case of a head sea,

$$C_{L_T} = C_{L_s} + aB \sin(\gamma' t + \phi) \quad , \tag{24}$$

where

$$\gamma' = \frac{2\pi}{\lambda}(V + c) \tag{25}$$

is the circular frequency of encounter for head seas.

The wake disturbance produced by the hydrofoil operating in waves is then

$$\zeta_w = -\left[C_{L_s} + aB \sin(\gamma t - \phi)\right] c' \, e^{-gh/V^2} \sin\frac{gs}{V^2} \tag{26}$$

for a following sea, and

$$\zeta_w = -\left[C_{L_s} + aB \sin(\gamma' t + \phi)\right] c' \, e^{-gh/V^2} \sin\frac{gs}{V^2} \tag{27}$$

for a head sea.

With the mathematical development of the disturbance wave completed, it is now possible to determine the water surface shape at any distance aft of the foil, which will then represent the wave pattern in which the second foil of a tandem system will be operating. At a distance ℓ aft of the quarter-chord point of the foil ($x = -\ell$), the original sea wave pattern will be represented, at time t, by

$$\eta = a \sin(\gamma t - \psi) \tag{28}$$

for a following sea, and by

$$\eta = a \sin(\gamma' t - \psi) \tag{29}$$

for a head sea, where

$$\psi = \frac{2\pi\ell}{\lambda} \quad . \tag{30}$$

However, interest is centered on the resultant wave at the time $t + \ell/V$, since at this time the disturbance created by the foil at time t is felt in the rear ($x = -\ell$), because of the time lag effect. Thus, at time $t + \ell/V$, the resultant wave surface at $x = -\ell$ in a following sea is represented by

$$\eta_r = a \sin\left(\gamma t + \frac{\gamma \ell}{V} - \psi\right) + \zeta_w \quad , \tag{31}$$

where ζ_w is the wave amplitude at $s = \ell$ due to a disturbance produced at time t. In this derivation, it is implicitly assumed that the existing sea wave pattern and the disturbance wave are superposable and that no interference exists between them. The resultant wave pattern at $x = -\ell$ in a following sea may now be expressed as

$$\eta_r = C \sin(\gamma t + \delta) - C_{L_s} c' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \quad , \tag{32}$$

where

$$\left.\begin{aligned} C &= a\sqrt{1 + B^2 c'^2 e^{-2gh/V^2} \sin^2 \frac{g\ell}{V^2} - 2Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \cos\left(\frac{\gamma\ell}{V} - \psi + \phi\right)} \\ \delta &= \tan^{-1} \frac{\sin\left(\frac{\gamma\ell}{V} - \psi\right) + Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \sin\phi}{\cos\left(\frac{\gamma\ell}{V} - \psi\right) - Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \cos\phi} \end{aligned}\right\} \tag{33}$$

Similarly, the resultant wave shape at $x = -\ell$ in a head sea is expressed as

$$\eta_r = D \sin(\gamma' t + \sigma) - C_{L_s} c' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \quad , \tag{34}$$

where

$$\left.\begin{aligned} D &= a\sqrt{1 + B^2 c'^2 e^{-2gh/V^2} \sin^2 \frac{g\ell}{V^2} - 2Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \cos\left(\frac{\gamma'\ell}{V} - \psi - \phi\right)} \\ \sigma &= \tan^{-1} \frac{\sin\left(\frac{\gamma'\ell}{V} - \psi\right) - Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \sin\phi}{\cos\left(\frac{\gamma'\ell}{V} - \psi\right) - Bc' e^{-gh/V^2} \sin \frac{g\ell}{V^2} \cos\phi} \end{aligned}\right\} \tag{35}$$

DETERMINATION OF DOWNWASH ANGLE ϵ AND OF $d\epsilon/d\alpha$

The angle of attack at the rear foil of a tandem hydrofoil system is dependent upon the mean downwash angle, ϵ. The sense of ϵ is positive when it tends to reduce the angle of attack at the rear foil. It is determined from the slope of the sinusoidal disturbance wave generated by the bound vortex of the forward foil. The downwash angle ϵ is the flow angle at the rear foil due to the disturbance wave created by the forward foil. Thus, it is not to be considered as being due to the influence of the orbital velocity of the sea wave profile above the rear foil; it is found from the expression for ζ_w and not η_r.

For a hydrofoil in waves, the disturbance wave is represented by equations (26) and (27) for following seas and head seas, respectively. The wave slope at the surface is then given as $d\zeta_w/ds$; the angle of downwash at a distance s behind the foil and at a depth h' below the water surface is expressed as

$$\epsilon = -\frac{d\zeta_w}{ds} e^{-gh'/V^2}$$

$$= \left[C_{L_s} + \alpha B \sin(\gamma t - \phi)\right]\frac{gc'}{V^2} e^{-g(h+h')/V^2} \cos\frac{gs}{V^2} \qquad (36)$$

for a following sea, and by

$$\epsilon = \left[C_{L_s} + \alpha B \sin(\gamma' t + \phi)\right]\frac{gc'}{V^2} e^{-g(h+h')/V^2} \cos\frac{gs}{V^2} \qquad (37)$$

for a head sea.

By using the preceding expressions for ϵ, the rate of change of downwash, $d\epsilon/d\alpha$, can be easily found. The terms in the expressions for ϵ that are functions of α are C_{L_s}, B, and ϕ: the derivative $\partial C_{L_s}/\partial\alpha$ is given by equation (7) and the derivatives $\partial B/\partial\alpha$ and $\partial\phi/\partial\alpha$ are given below as

$$\frac{\partial B}{\partial \alpha} = \frac{1}{2B}\frac{\partial}{\partial \alpha}\left(\frac{\partial C_{L_s}}{\partial h}\right)^2$$

$$= \frac{4\pi^2\alpha}{B}\left\{\frac{\pi A + 8h/c'}{\pi A + (8h/c') + 4\pi P_1}\frac{\partial P_1}{\partial h} + P_1\frac{(32\pi P_1/c') - 4\pi\frac{\partial P_1}{\partial h}(\pi A + 8h/c')}{[\pi A + (8h/c') + 4\pi P_1]^2}\right\}^2 \qquad (38)$$

and

$$\frac{\partial \phi}{\partial \alpha} = \frac{1}{1 + \tan^2\phi} \frac{-\tan \phi}{\left(\frac{\partial C_{L_s}}{\partial h}\right)} \frac{\partial}{\partial \alpha}\left(\frac{\partial C_{L_s}}{\partial h}\right) = \frac{-\tan \phi}{(1 + \tan^2\phi)a} \quad . \tag{39}$$

The value of $d\epsilon/d\alpha$ for a hydrofoil in a following sea is then

$$\frac{d\epsilon}{d\alpha} = \left[\frac{\partial C_{L_s}}{\partial \alpha} + a\frac{\partial B}{\partial \alpha}\sin(\gamma t - \phi) - aB\frac{\partial \phi}{\partial \alpha}\cos(\gamma t - \phi)\right]\frac{gc'}{V^2}\, e^{-g(h+h')/V^2} \cos\frac{gs}{V^2}$$

$$= \left[\frac{\partial C_{L_s}}{\partial \alpha} + a\sqrt{\left(\frac{\partial B}{\partial \alpha}\right)^2 + B^2\left(\frac{\partial \phi}{\partial \alpha}\right)^2}\sin(\gamma t - \phi - \vartheta)\right]\frac{gc'}{V^2}\, e^{-g(h+h')/V^2} \cos\frac{gs}{V^2} \quad , \tag{40}$$

where

$$\vartheta = \tan^{-1}\frac{B\frac{\partial \phi}{\partial \alpha}}{\frac{\partial B}{\partial \alpha}} \quad . \tag{41}$$

Similarly, for the hydrofoil in a head sea,

$$\frac{d\epsilon}{d\alpha} = \left[\frac{\partial C_{L_s}}{\partial \alpha} + a\sqrt{\left(\frac{\partial B}{\partial \alpha}\right)^2 + B^2\left(\frac{\partial \phi}{\partial \alpha}\right)^2}\sin(\gamma' t + \phi + \vartheta)\right]\frac{gc'}{V^2}\, e^{-g(h+h')/V^2} \cos\frac{gs}{V^2} \quad . \tag{42}$$

The change in angle of attack at the rear foil of a restrained tandem hydrofoil system in waves is given by

$$\Delta\alpha = \hat{w}_o - \epsilon \quad , \tag{43}$$

where $\hat{w}_o$ and ϵ are evaluated at the location of the foil and are functions of the wave properties.

CONCLUSIONS

The equations developed in the present report give expressions for the downwash angle and its rate of change with foil angle, for a hydrofoil moving under a free surface disturbed by sea waves. The total wave disturbance is assumed to be a linear superposition of the effects of the quasi-steady lift and the time-dependent sinusoidal lift variation due to orbital motion and change of submergence. The resultant downwash angle, ϵ, and its rate of change, $d\epsilon/d\alpha$, are then the sum of the contribution found for the case of motion under an undisturbed free surface (Reference 1) and a sinusoidal time-dependent component due to the sea wave influence. The phase angles for the time-dependent components of ϵ and $d\epsilon/d\alpha$ (relative to the phase of the disturbing sea waves) are negatives of each other for the following-sea and head-sea conditions. The frequencies for the time-dependent terms in ϵ and $d\epsilon/d\alpha$ are, however, fundamentally different for the following sea and head sea.

REFERENCES

1. Hugli, W.C., Jr. and Kaplan, Paul. *Theoretical Analysis of the Longitudinal Stability of a Tandem Hydrofoil System in Smooth Water.* Stevens Institute of Technology, Experimental Towing Tank Report No. 479, July 1953. CONFIDENTIAL

2. Keldysch, M.V. and Lavrentiev, M.A. *On the Motion of an Aerfoil Under the Surface of a Heavy Fluid, i.e. a Liquid.* Science Translation Service, Cambridge, Mass., STS-75, November 1949.

3. Korvin-Kroukovsky, B.V. and Wernick, R.J. *A Note on the Lift and Drag of Hydrofoils -- Application of Theory to Experimental Results, Part I: Application of Thin-Wing Theory.* Stevens Institute of Technology, Experimental Towing Tank Note No. 136, January 1951. CONFIDENTIAL

4. Michel, W.H. and Lavagnino, A. *Sea Conditions and Their Influence on Hydrofoil Lift.* Bath Iron Works Corporation, by Gibbs & Cox, Inc., Technical Report No. 7, August 1952. CONFIDENTIAL

5. Meyer, R.X. *Surface Waves Produced by Hydrofoils.* Hydrofoil Corporation, Technical Memorandum No. HM-00, April 1950. CONFIDENTIAL